2. For each matrix A, find a basis for each generalized eigenspace of L_A consisting of a union of disjoint cycles of generalized eigenvectors. Then find a Jordan canonical form J of A.

(a) $A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$ (b) $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$

(c) $A = \begin{pmatrix} 11 & -4 & -5 \\ 21 & -8 & -11 \\ 3 & -1 & 0 \end{pmatrix}$ (d) $A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 1 & -1 & 3 \end{pmatrix}$

3. For each linear operator T, find a basis for each generalized eigenspace of T consisting of a union of disjoint cycles of generalized eigenvectors. Then find a Jordan canonical form J of T.

(a) T is the linear operator on $\mathsf{P}_2(R)$ defined by $\mathsf{T}(f(x)) = 2f(x) - f'(x)$

(b) V is the real vector space of functions spanned by the set of real valued functions $\{1, t, t^2, e^t, te^t\}$, and T is the linear operator on V defined by $\mathsf{T}(f) = f'$.

(c) T is the linear operator on $\mathsf{M}_{2\times 2}(R)$ defined by $\mathsf{T}(A) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot A$ for all $A \in \mathsf{M}_{2\times 2}(R)$.

(d) $\mathsf{T}(A) = 2A + A^t$ for all $A \in \mathsf{M}_{2\times 2}(R)$.

4.[†] Let T be a linear operator on a vector space V, and let γ be a cycle of generalized eigenvectors that corresponds to the eigenvalue λ. Prove that span(γ) is a T-invariant subspace of V.

5. Let $\gamma_1, \gamma_2, \ldots, \gamma_p$ be cycles of generalized eigenvectors of a linear operator T corresponding to an eigenvalue λ. Prove that if the initial eigenvectors are distinct, then the cycles are disjoint.

6. Let T: V → W be a linear transformation. Prove the following results.

(a) N(T) = N(−T).

(b) $\mathsf{N}(\mathsf{T}^k) = \mathsf{N}((-\mathsf{T})^k)$.

(c) If V = W (so that T is a linear operator on V) and λ is an eigen- ... integer k

2. Let T be a linear operator on a finite-dimensional vector space V such that the characteristic polynomial of T splits. Suppose that $\lambda_1 = 2$, $\lambda_2 = 4$, and $\lambda_3 = -3$ are the distinct eigenvalues of T and that the dot diagrams for the restriction of T to K_{λ_i} $(i = 1, 2, 3)$ are as follows:

$\lambda_1 = 2$			$\lambda_2 = 4$		$\lambda_3 = -3$	
•	•	•	•	•	•	•
•	•		•			
•			•			

Find the Jordan canonical form J of T.

3. Let T be a linear operator on a finite-dimensional vector space V with Jordan canonical form

$$\left(\begin{array}{ccc|cc|cc} 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{array}\right).$$

(a) Find the characteristic polynomial of T.
(b) Find the dot diagram corresponding to each eigenvalue of T.
(c) For which eigenvalues λ_i, if any, does $E_{\lambda_i} = K_{\lambda_i}$?
(d) For each eigenvalue λ_i, find the smallest positive integer p_i for which $K_{\lambda_i} = N((T - \lambda_i I)^{p_i})$.
(e) Compute the following numbers for each i, where U_i denotes the restriction of $T - \lambda_i I$ to K_{λ_i}.
  (i) $\text{rank}(U_i)$
  (ii) $\text{rank}(U_i^2)$
  (iii) $\text{nullity}(U_i)$
  (iv) $\text{nullity}(U_i^2)$

4. For each of the matrices A that follow, find a Jordan canonical form J and an invertible matrix Q such that $J = Q^{-1}AQ$. Notice that the matrices in (a), (b), and (c) are those used in Example 5.

(a) $A = \begin{pmatrix} -3 & 3 & -2 \\ -7 & 6 & -3 \\ 1 & -1 & 2 \end{pmatrix}$ **(b)** $A = \begin{pmatrix} 0 & 1 & -1 \\ -4 & 4 & -2 \\ -2 & 1 & 1 \end{pmatrix}$

(c) $A = \begin{pmatrix} 0 & -1 & -1 \\ -3 & -1 & -2 \\ 7 & 5 & 6 \end{pmatrix}$ **(d)** $A = \begin{pmatrix} 0 & -3 & 1 & 2 \\ -2 & 1 & -1 & 2 \\ -2 & 1 & -1 & 2 \\ -2 & -3 & 1 & 4 \end{pmatrix}$